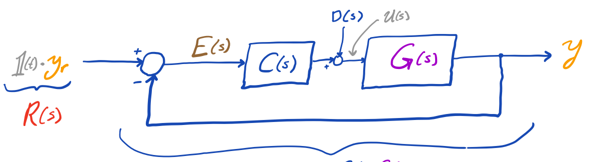
Feedback Control



TF: $\dfrac{C(s)\,G(s)}{1 + C(s)\,G(s)}$

- PI Controller: $C(s) = K\left[1 + \frac{1}{T_I s}\right]$

$$\begin{bmatrix} E(s) \\ G(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{1+CG} & \frac{-G}{1+CG} \\ \frac{C}{1+CG} & \frac{1}{1+CG} \end{bmatrix} \begin{bmatrix} R(s) \\ D(s) \end{bmatrix} \Rightarrow E(s) = \frac{1}{1+CG} R(s) \Big|_{D(s)=0}$$

Theorem: Sys is BIBO stable iff:

1. $\frac{1}{1+CG}$ has all poles $\in$ OLHP
2. $\frac{1}{1+CG}$ has no p/z cancellations on j-axis or ORHP.

$$\begin{bmatrix} d \\ r \end{bmatrix} \rightarrow sys \rightarrow \begin{bmatrix} e \\ u \end{bmatrix}$$

BIBO

$\boxed{\text{"Standing assumption" on } C(s)}$

Theorem: If standing ass. + $d(t) = 0$: $\boxed{\text{Poly}^{\underline{n}} \text{ reference}}$

for $r(t) = \sum_{j=0}^{k-1} c_j t^j \cdot \mathbb{1}(t)$ (★) poly$^{\underline{n}}$ order $k-1$

$e(\infty) = 0$ if $C(s)\,G(s)$ has $\geq k$ poles @ origin.

$\boxed{\text{Theorem}}$: Suppose $\boxed{\text{Poly}^{\underline{n}} \text{ disturb}}$

1. Standing ass.
2. $r(t) = \text{poly}^{\underline{n}}(k-1)$
3. $d(t) = \text{poly}^{\underline{n}}(j-1),\ j \leq k$

Then $e(\infty) = 0$

if
1. CG has $\geq k$ poles @ $s=0$
2. j poles are from $C(s)$

Internal Model Principal

- $R(s), D(s)$ = strictly proper rational, poles on j-axis.
- $C(s)$ solves tracking problem iff:
  1. Standing ass.
  2. $C \cdot G(s)$ have same poles as $R, D(s)$ w/ same multiplicity.
  3. $C(s)$ has all poles of D w/ same mult.

(★) Unsolvable if zeros(G) = any poles(R, D)

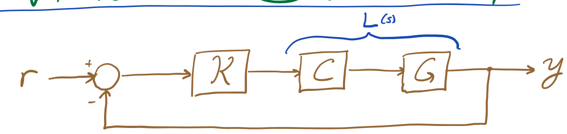
Principal of Argument

Given: $G(s)$, path D
↳ No self-intersection
↳ Clockwise, closed

- $G(s)$ has n poles, m zeros in D

$\Rightarrow \mathcal{G} = G(D)$ encircles origin $n-m$ times CCW

Nyquist Stability



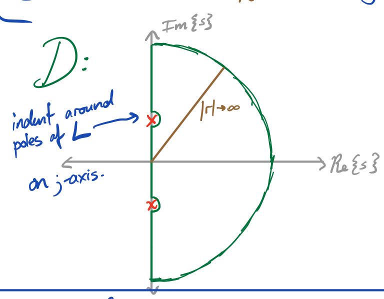
Criterion:
1. L is strictly proper
2. L has n poles in ORHP
3. No unstable p-z cancellations.

Closed-Loop is BIBO Stable iff:

$\mathcal{L} = \{L(s) \mid s \in D\}$ satisfies:

1. Not pass thru $\frac{-1}{K}$
2. Encircles $s = \frac{-1}{K}$ exactly n times CCW.

D:



Stability Margins

- Distance b/w $\mathcal{L} \longleftrightarrow \frac{-1}{K} = 1$ is measure of stability.
- ω_c: $|L(\omega_c j)| = 1$
- $\bar{\omega}$: $\angle L(\bar{\omega} j) = -\pi$

Gain Margin $GM = \dfrac{1}{|L(j\bar{\omega})|}$

Phase Margin $PM = \angle L(j\omega_c) - (-\pi)$

Nyquist Tricks

- Only really need $L(j\omega)$:
  ↳ $L(-j\omega) = \left(L(-j\omega)\right)^*$
  ↳ Semicircle $\xrightarrow{\text{↳}}$ origin for strictly proper $L(s)$.
- Phasor manipulation: $z_i = r_i \angle \phi_i$

$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2 \qquad z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$

★ Multiply by denominator conjugate to find phase overall.

BODE PLOTS

- Frequency response of <u>open loop</u>:

$$\begin{cases} |L|_{DB} \\ \angle L \end{cases} \text{ vs. } \log(\omega)$$

- $|L|_{DB} \triangleq 20 \log |L|$
  - ↳ Simply add components freq responses!
- Let $L(s) = K \dfrac{(s+z_1)(s+z_2)\cdots}{(s+p_1)(s+p_2)\cdots}$

$$= \underbrace{K\frac{z_1 z_2 \cdots}{p_1 p_2 \cdots}}_{K_0} \frac{\left(\frac{s}{z_1}+1\right)\left(\frac{s}{z_2}+1\right)\cdots}{\left(\frac{s}{p_1}+1\right)\left(\frac{s}{p_2}+1\right)\cdots}$$

$$\Longrightarrow 20\log\left(L(j\omega)\right) = \underbrace{|K_0|_{DB} + \left|\frac{s}{z_1}+1\right|_{DB} + \left|\frac{s}{z_2}+1\right|_{DB} \cdots - \left|\frac{s}{p_1}+1\right|_{DB} - \left|\frac{s}{p_2}+1\right|_{DB}}_{\text{Linear Combo!}}$$

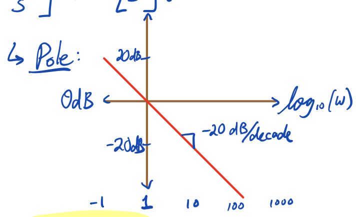
PM, GM FROM BODE

- $\omega_c : |L|_{DB} = 0dB$
- $\bar{\omega} : \angle L(j\omega) = \pm\pi$
- PM: Distance from $\angle L(\omega_c j) \longleftrightarrow -\pi$
- GM: $GM_{DB} = -\left|L(j\bar{\omega})\right|_{DB}$

① $K_0 : \begin{cases} 20\log_{10}(K_0) & \text{magnitude (offset)} \\ \angle = \begin{cases} 0 & \text{for } K_0 > 0 \\ -\pi & \text{for } K_0 < 0 \end{cases} \end{cases}$

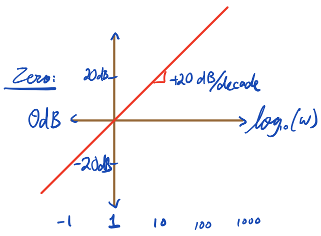
② $\left[\frac{1}{s}\right]$ or $[s]$:

↳ <u>Pole</u>:



$\angle \frac{1}{j\omega} = \frac{-\pi}{2}$

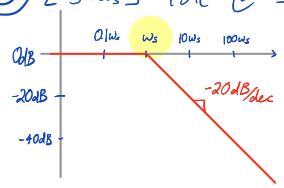
<u>Zero</u>:

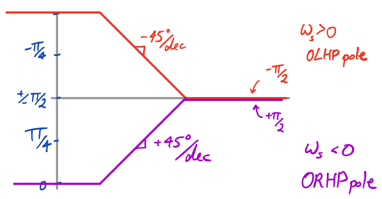


$\angle j\omega = \frac{\pi}{2}$

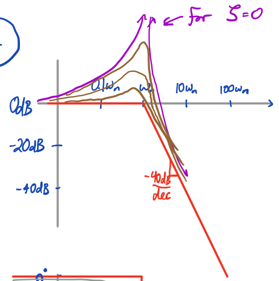
③ $\left[\frac{1}{s+\omega_s}\right]$: Pole @ $s = -\omega_s$

$[s+\omega_s]$: Flipped!





④



$$\boxed{L(s) = \frac{1}{1 + \frac{2\zeta}{\omega_n} j\omega + \left(\frac{j\omega}{\omega_n}\right)^2}}$$

↳ <u>Flipped</u> for $L(s) = 1 + \frac{2\zeta}{\omega_n} j\omega + \left(\frac{j\omega}{\omega_n}\right)^2$