CHE260 Part 2 Abridged

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Chapter 1

Basics of Heat Transfer

1.1 Introduction

Heat is energy transferred because of temperature differences. The modes of heat transfer include:

1. Conduction
2. Convection
3. Radiation

1.2 Conduction

Fourier's Law of Heat Conduction:

$$\dot{Q} = -kA\frac{\Delta T}{\Delta x}$$

1. k: THERMAL CONDUCTIVITY. It's really a function of temperature but we assume it's constant in this course.
2. A: Area of surface

DIFFUSIVITY: $\alpha = \frac{k}{\rho c_p}$ is the rate of heat propagation through a medium by volume. Small $\rightarrow$ mostly absorbed, not transmitted.

1.3 Convection

Heat transfer from solid to fluid in motion $\dot{Q}_{conv} = \dot{Q}_{cond} + \dot{Q}_{fluidmotion}$

NEWTON'S LAW OF COOLING:

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty$$

Where h is experimentally and subjectively determined.

1.4 Radiation

$$\dot{Q}_{emit} = \varepsilon\sigma A_s T_s^4$$

1. ε: Emissivity
2. σ: Stefan-Boltzmann constant

$$\dot{Q}_{rad,net} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4)$$

$$\dot{Q}_{conv+rad} = h_{combined}A_s(T_s - T_\infty)$$

Chapter 2

Steady Heat Conduction

2.1 Thermal Resistance Networks

$$\dot{Q} = \frac{T_{\infty,1} - T_{\infty,2}}{R_{total}}$$

$V = IR \rightarrow \Delta T = \dot{Q}R$

Resistances add in the same ways as electrical resistances. For a wall, you must account for convective resistances on both sides and the internal conductive resistance.

Thermal contact resistance: Results from microscopic gaps between two objects in physical contact. R_c is the resistance per unit area of contact. Use the ratio of $R_{layer} : R_c$ to determine if the contact resistance is worth taking into consideration.

2.2 Heat Conduction with Cylinders and Spheres

2.2.1 Cylinders

By integrating $\dot{Q}_{cond,cyl} = -kA\frac{dT}{dr}$ we get

$$R_{cyl} = \frac{1}{2\pi Lk} \ln(r_2/r_1)$$

Where r_2 is outer, r_1 is inner, L is length, etc.

2.2.2 Spheres

By the same logic:

$$R_{sph} = \frac{r_2 - r_1}{4\pi r_1 r_2 k}$$

For both spheres and cylinders: you must figure in $R_{conv,in} + R_{cyl} + R_{conv,out}$

2.3 Critical Radius of Insulation

For flat walls, more insulation $\rightarrow$ more thermal resistance.

$$r_{cr,cyl} = \frac{k}{h}$$
$$r_{cr,sph} = \frac{2k}{h}$$

2.4 Chapter 10 Class Notes

- Biot number: Ratio of convection : conduction $= Bi = \frac{hw}{k}$, w is the width of the surface.
- Diffusion time: $t_D = \frac{L^2}{\alpha}$
- Rate Limiting Step: If $Bi < 1$: CONVECTION. Else, conduction.
- Governing Conduction Equation:

$$\rho C \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \dot{S}$$

  - ρ = density
  - C = heat capacity
  - k = thermal CONDUCTIVITY
  - $\dot{S}$ = heat supplied per second

2.5 Fins

Fins allow you to increase the area of convection, leading to greater $\dot{Q}_{conv}$. We analyze fins at steady state with constant k, h.

FIN EQUATION: For an infinitesimal slice of a fin, we have:

$$\dot{Q}_{cond,x} = \dot{Q}_{cond,x+\Delta x} + \dot{Q}_{conv}$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

Where $m^2 = \frac{h\rho}{kA_c}$, $\theta = T - T_\infty$

$$\theta(x) = c_1 e^{mx} + c_2 e^{-mx}$$

Boundary conditions: $\theta(0) = T_{base} - T_\infty$. Boundary condition 2 depends on situation...

Note that p is the perimeter of the fin at the tip.

- **Infinite Fin Length:** $T_{tip} = T_\infty$.
- **Insulated Tip:** $\dot{Q}_{tip} = 0 \rightarrow \frac{d\theta}{dx}|_{x=L} = 0$, so the general heat out is:

$$\dot{Q} = \sqrt{hpkA_c}(T_b - T_\infty)\tanh(mL)$$

- Convective and radiative heat loss. Use insulated tip length with corrected length:

$$L_c = L + A_c/p$$

$L_{c,rect} = L + t/2$, t = thickness. $L_{c,cyl} = L + D/4$.

FIN EFFICIENCY: $\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,max}}$.

$$\dot{Q}_{max} = hA_{fin}(T_b - T_\infty)$$

FIN EFFECTIVENESS:

$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{nofin}} = \frac{\dot{Q}_{fin}}{hA_b(T_b - T_\infty)}$$

$$\varepsilon_{fin} = \frac{A_{fin}}{A_b}\eta_f in$$

Proper Fin Length: Too long is a waste. $\frac{\dot{Q}_{fin}}{\dot{Q}_{longfin}} = tanmL$. Generally $mL = 1$ is a good compromise for 76.2% efficiency.

$$\dot{Q}_{fin} = \frac{T_b - T_\infty}{R}$$

$$R = \frac{1}{hA_{fin}\eta_{fin}}$$

Chapter 3

Transient Temperature Changes

3.1 Lump Analysis

Treating the body like it just has one internal temperature that is the same throughout. Used when $Bi = \frac{hL_c}{k} =\leq 0.1$, $L_c = V/A_s$.

$$hA_s(T_\infty - T)dt = mc_p dT$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = \exp(-bt)$$

Where $b = \frac{hA_s}{\rho V c_p}$

$$Q_{net} = mc_p[T(t) - T_i]$$

3.2 Transient Heat Conduction

We study 1-D transient heat conduction for walls, spheres, and cylinders.

- $\theta = \frac{T - T_\infty}{T_i - T_\infty}$ = dimensionless temperature
- $X = x/L$ = dimensionless time
- $Bi = hL/k$ = dimensionless heat transfer coefficient

The fourier series for each of these is on the formula sheet. When $\tau > 0.2$ we can use the single-term fourier approximation. $\tau = \frac{\alpha t}{L^2}$.

3.2.1 Illustrative Example:

How long until the centre of an egg is 70 Celsius?

1. $Bi = \frac{hr_0}{k}$. $Bi > 0.1 \rightarrow$ Transient 1-D analysis. Less means lump analysis.

2. Find λ_1 and A_1 from the table.

3. Find τ, the fourier number.

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

   to determine the fourier number τ. T_0 is the final temperature.

4. $t = \frac{\tau r_0^2}{\alpha}$ = time until middle reaches final temperature T_0.

Chapter 4

External Forced Convection

4.1 Physical Mechanisms of Convection

No Slip Condition: Fluid velocity is zero and 'sticks' to the surface of objects it flows by. Therefore, we have pure conduction at the surface of objects in fluids.

IMPORTANT NUMBERS:

1. **Nusslet** $Nu = \frac{hL_c}{k} = \frac{\dot{q}_{conv}}{\dot{q}_{cond}}$
2.
3.

4.2 Classifying Fluid Flows

- Viscous: internal stickiness is significant. Inviscous: internal stickiness is not relevant.
- **External** means flow is unbounded. **Internal** means flow is bounded on all sides (e.g. pipe).
- **Compressible** corresponds to $Ma > 0.3$ or $v > 100m/s$.
- **Natural** means that it's only stuff like buoyancy and convection.
- **Steady** means stable through time. **Uniform** means stable with location.
- **Transient** means developing steady flows. **Periodic** means oscillatory.

4.3 Velocity Boundary Layer

Consider fluid flowing over a surface. Due to the non-slip condition, the fluid at the surface doesn't move. δ is the height above the surface at which the x-velocity reaches $0.99v$ where v is the upstream velocity. It's the **boundary layer thickness**. It divides the 'boundary layer' from the 'irrotational flow' area.

$F/A = \tau$ which is shear stress.

$$\tau_s = \mu \frac{du}{dy}|_{y=0}$$

$\nu = \mu/\rho$ is kinematic viscosity. Measured in stokes (cm2/s)

More practically:

$$\tau_s = C_f \frac{\rho V^2}{2}$$

Chapter 5

In-Class Things and Guides

5.1 Thermal Boundary Layer

This is the same as the velocity boundary layer but for temperature. It's where $T_s - T = 0.99(T_\infty - T_s)$. Its size increases in the direction of flow. $\dot{Q}_x$ depends strongly on temperature gradient at the surface, so it's important!

Prantl Number is the relative thickness of the velocity : thermal layer.

$$Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$$

5.2 Laminar and Turbulent Flows

Reynold numbers dictate whether you are in laminar, turbulent, or transitional flow. If $Re > Re_{cr}$, we have turbulent flow.

5.3 Drag and Heat Transfer in External Flow

Lift is just the sum of forces that are perpendicular to flow.

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

There is a pressure component and a frictional component to drag force. Pressure dominates with large Reynold's numbers while friction dominates with small Reynold's numbers.

The area in that formula for drag coefficient is the frontal projection of the object for most objects, but is the actual area of a flat plate that is in line with the flow.

5.3.1 Heat Transfer

Heat transfer relies on the same variables as drag.

$$Nu = C_{onst} Re_i^m Pr^n$$

where m, n are constant integer exponents and C is a constant that results from the geometry.

Since C_D, h vary along the surface, we want the AVERAGE values of them.

$x_c r$ is the x coordinate where flow becomes turbulent over a plate. $Re_{cr} = 5 \times 10^5$ is the generally accepted way to get the critical reynold's number.

Friction Coefficients at x:

- Laminar:

$$\delta_{v,x} = \frac{4.91x}{Re_x^0.5}$$

$$C_{fx} = \frac{0.664}{Re_x^0.5}$$

$$Nu_x = \frac{h_x x}{k} = 0.332 Re_x^{0.5} Pr^{1/3}$$

- Turbulent:

$$\delta_{v,x} = \frac{0.38x}{Re_x^{1/5}}$$

$$C_{fx} = \frac{0.059}{Re_x^{1/5}}$$

$$Nu_x = \frac{h_x x}{k} = 0.0296 Re_x^{0.8} Pr^{1/3}$$

Average Friction Coefficients:

- Laminar:

$$C_f = \frac{1.33}{Re_L^{0.5}}$$

- Turbulent:

$$C_f = \frac{0.074}{Re_L^{1/5}}$$

- Laminar AND Turbulent:

$$C_f = \frac{0.74}{Re_L^{1/5}}$$

Rough, Turbulent Surfaces:

$$C_f = (1.89 - 1.62 \log \frac{\varepsilon}{L})^{-2.5}$$

Average Heat Transfer Coefficients

- Laminar: $Nu_{avg} = \frac{hL}{k} = 0.664 Re_L^0.5 Pr^{1/3}$
- Turbulent: $Nu_{avg} = \frac{hL}{k} = 0.037 Re_L^{0.8} Pr^{1/3}$
- Entire Plate: $Nu_{avg} = \frac{hL}{k} = (0.037 Re_l^{0.8} - 871) Pr^{1/3}$

There's some substantial deviation from these if the fluid has very niche characteristics (small Pr for liquid metals, etc.). Here is a formula that applies to all fluids:

$$Nu_x = \frac{h_x x}{k} = \frac{0.3387 Pr^{1/3} Re_x^{0.5}}{[1 + (0.0468/Pr)^{2/3}]^{1/4}}$$

5.3.2 Unheated Starting Length

It's common for the first bit of the plate that the fluid goes over to not be heated. Let Ξ be the distance from the physical and heated starting points.

- Laminar:
$$Nu_x = \frac{Nu_{x,(\Xi=0)}}{[1 - (\Xi/x)^{3/4}]^{1/3}}$$
$$h_{avg} = \frac{2[1 - (\Xi/x)^{3/4}]}{1 - \Xi/L} h_{x=L}$$
- Turbulent:
$$Nu_x = \frac{Nu_{x,(\Xi=0)}}{[1 - (\Xi/x)^{9/10}]^{1/9}}$$
$$h_{avg} = \frac{5[1 - (\Xi/x)^{9/10}]}{4(1 - \Xi/L)} h_{x=L}$$

5.3.3 Uniform Heat Flux

When the plate has uniform heat flux instead of uniform heat. Relations for $\Xi \neq 0$ apply here too.

- Laminar:
$$Nu_x = 0.453 Re_x^{0.5} Pr^{1/3}$$

- Turbulent:
$$Nu_x = 0.0308 Re_x^{0.8} Pr^{1/3}$$

$$\dot{Q} = \dot{q}_s A$$
$$\dot{q}_s = h_x[T_s(x) T_\infty] \rightarrow T_s(x) = T_\infty + \frac{\dot{q}_s}{h_x}$$